EXHIBIT 8.9 Data for Valuation using Sales or Market Multiples

(in millions)	PG	KMB	JNJ	CL
Operating value	$ —	$51,966	$268,717	$65,514
Sales	$74,035	$18,880	$ 70,517	$16,356
R&D Expenditures	$ 2,000	$ 368	$ 8,817	$ 277
NOA	$84,538	$ 7,988	$ 55,408	$ 6,262
NFL (*NFA*)	$23,318	$ 7,934	$(16,146)	$ 6,007
Common shares outstanding	2,721 shares	363 shares	2,767 shares	897 shares

Valuation Using a Sales Multiple

We use the Operating Value-to-Revenue multiple for PG and its comparable company KMB to illustrate the use of a sales multiple to estimate value. Note that the focus on sales should be done with some caution. If, for example, the company we are valuing has a different cost structure than its comparable companies, this will add error to our estimation. Again note that we use the sales multiple to compute an estimate of operating value rather than equity value. Use of Operating Value-to-Revenue would be expected to be superior to Equity Value-to-Sales since the sales revenues are independent of financial activities. Sales is the driver of value available to both debt and equity holders so the Operating Value-to-Revenue multiple is more appropriate than the Equity Value-to-Sales ratio though **finance.yahoo.com** shows both in **Exhibit 8.1**.

The Operating Value-to-Revenue ratio for KMB is 2.75 computed as $51,966/$18,880. The average multiple from the three comparable companies is 3.52. Applying this multiple to PG, we estimate operating value at $260,603. This implies an equity value of $237,285 after subtracting *NFL* yielding an estimated price per share of $87.21. This stock price estimate suggests that PG stock was undervalued based on a $77.83 closing price at December 5, 2015.

Finally, note that sometimes the sales multiple is used rather than an income-based multiple when the company being valued (and, perhaps, also the comparable companies) is reporting a loss since a multiple of a loss is meaningless. Such a justification should be viewed with skepticism; the fact that the firm is losing money may in and of itself lead us to question the wisdom of investing in the firm!

Valuation Using the Ratio of (Price–Book) to R&D

To the extent that, for a company with a large amount of R&D, the difference between the market value of equity and the book value of equity may be seen as the market's valuation of R&D, the ratio of (Price – Book)/R&D may be a valid multiple for valuation. In firms where this is possibly so, (Price − Book) is often viewed as composed primarily of unrecorded assets arising because of the expensing (rather than capitalization) of R&D. If we were examining a firm reporting under IFRS, such as Unilever, we may need to revisit this assumption since IFRS capitalizes development costs. We illustrate the use of this ratio for PG.

Since R&D is related to operating activities, the use of price and book value of *equity* appears odd. However, given the common assumption that the book value and market value of *NFL* are equal this is not a problem. This can be seen as:

$$\begin{aligned} V_{Eq} - BV_{Eq} &= (V_{Ent} - V_D) - (\boxed{NOA} - NFL) \\ &= V_{Ent} - V_D - \boxed{NOA} + NFL \\ &= (V_{Ent} - \boxed{NOA}) - (V_D - NFL) \\ &= (V_{Ent} - \boxed{NOA}) - 0 \\ &= V_{Ent} - \boxed{NOA} \end{aligned}$$

That is, if the market and book value of *NFL* are equal, the premium is the same for enterprise operations and equity.[12] Thus, we will calculate the ratio using operating numbers as that is our primary focus, but we are reminded that the multiple is unchanged (but the value estimate does change) if the market value and book value of financial activities are equal. If the assumption that the market value and book value of financial activities are equal is not appropriate, there is yet another reason why the calculation should focus on operating value rather than equity value.

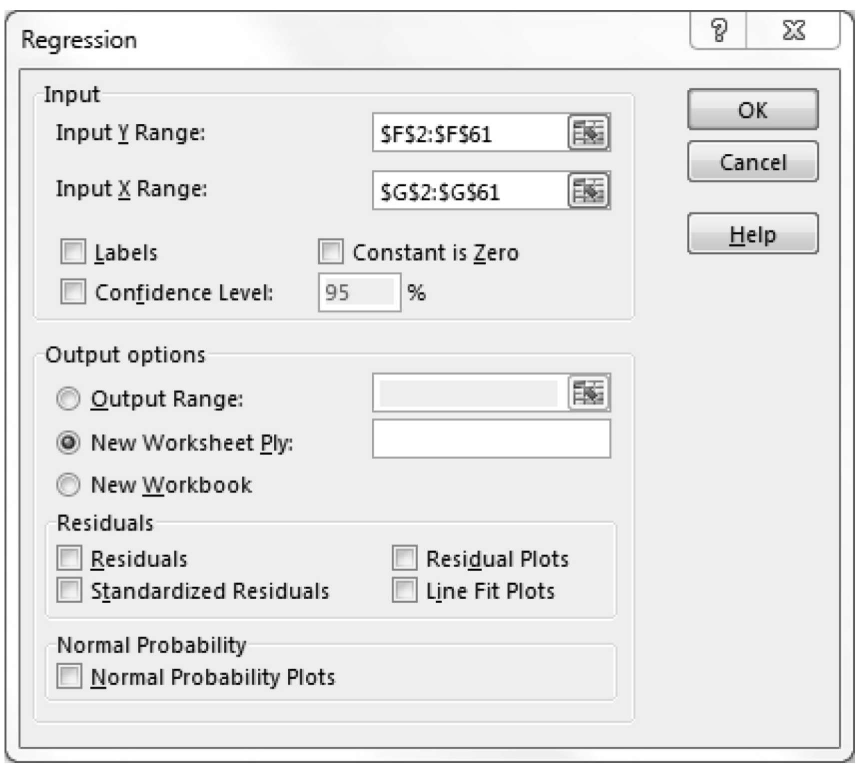
To compute an estimate of value based on the (P − B)/R&D ratio, we begin with a calculation of this ratio for KMB. For KMB, this ratio is 119.51 computed as ($51,966 − $7,988)/$368. To obtain the estimate of operating value for PG, we utilize the average multiple for the comparable companies of 119.20 and calculate value of $322,938 as 119.20 × $2,000 + $84,538. This implies an equity value of $299,620 after subtracting *NFL* yielding an estimated price per share of $110.11. The $110.11 stock price estimate suggests that PG stock was underpriced based on a $77.83 closing price.

[12] Clearly, the value of the interest tax shield is ignored when this assumption is made.

EXHIBIT 9A.7

	A	B	C	D	E	F	G
1	Date	r_{PG}	r_{rf}	r_{Mkt}		$r_{PG} - r_{rf}$	$r_{Mkt} - r_{rf}$
2	Nov-15	0.03995	(0.00556)	0.00542		0.04551	0.01098
3	Oct-15	(0.02016)	(0.00654)	0.00050		(0.01363)	0.00704
4	Sep-15	0.07126	(0.00887)	0.08298		0.08013	0.09186
5	Aug-15	0.01797	0.01380	(0.02644)		0.00417	(0.04025)
6	Jul-15	(0.07862)	0.00049	(0.06258)		(0.07911)	(0.06307)
7	Jun-15	(0.01166)	0.01279	0.01974		(0.02445)	0.00695
8	May-15	(0.00191)	(0.02321)	(0.02101)		0.02129	0.00219
9	Apr-15	(0.01409)	(0.00479)	0.01049		(0.00930)	0.01528
10	Mar-15	(0.02185)	(0.01092)	0.00852		(0.01093)	0.01944
11	Feb-15	(0.03747)	0.00669	(0.01740)		(0.04416)	(0.02409)
12	Jan-15	0.00997	(0.03160)	0.05489		0.04157	0.08649
13	Dec-14	(0.06807)	0.04977	(0.03104)		(0.11784)	(0.08081)
14	Nov-14	0.00730	0.00235	(0.00419)		0.00495	(0.00654)
15	Oct-14	0.03621	0.01388	0.02453		0.02233	0.01065

EXHIBIT 9A.8



interval, we are reminded that this is an estimate. Further, had we used a different proxy for the risk-free or market rate of return, we would have obtained a different estimate of β. Additional factors that could have altered our estimate of β are the use of weekly rates of returns or less than 60 observations. In short, we must always remember that β is an estimate. It may be worth checking the reasonableness of the estimated β against those shown by other sources such as Bloomberg or financial websites.

One other item to address regarding the output in **Exhibit 9A.9** is the estimate of the intercept contained in cell B17. This is the average monthly rate of return not captured by the relation between the excess return on PG and the excess return on the market; that is, PG's rate of return not captured by β. This is referred to as an alpha return (α). Some investors specifically seek out stocks with positive

EXHIBIT 10.3 PG Multiyear Forecasts of Sales Growth, *PM*, and *ATO*

	2016E	2017E	2018E	2019E	2020E
Sales growth.	1.80%	2.00%	2.20%	2.30%	2.40%
Operating *PM*	13.93%	13.73%	13.35%	13.20%	12.92%
Operating *ATO*	0.858	0.836	0.817	0.805	0.807

These assumptions of sales growth, profit margin, and asset turnover are in line with results seen in recent years and appear to reflect reasonable expectations of future outcomes. Further information on the computations supporting these assumptions and the relation to current accounting information were discussed in Module 7.

Using the above inputs, we can forecast expectations of PG's future sales, *NOPAT* and *NOA*. Forecasted sales for each year are the prior year sales multiplied by (one-plus) the expected sales growth rate, or 1.018 in the case of computing 2016 sales. *NOPAT* is computed using forecasted sales each year times the forecasted operating profit margin (*PM*); *NOA* is computed using forecasted sales of the following year divided by the forecasted operating asset turnover (*ATO*). As a reminder, the 2016 *NOA* is the amount of forecasted resources necessary to support the 2017 Sales. Thus, 2016 *NOA* is computed as the 2017 Sales divided by the 2017 *ATO* forecast of 0.836. Forecasted numbers for 2016 through 2020 are once again shown in **Exhibit 10.4**.

EXHIBIT 10.4 PG Multiyear Forecasts of Sales, *NOPAT*, and *NOA* ($ millions)

	2015	2016 Est.	2017 Est.	2018 Est.	2019 Est.	2020 Est.
Sales.	$76,279	$77,652 ($76,279 × 1.018)	$79,205 ($77,652 × 1.020)	$80,948 ($79,205 × 1.022)	$82,809 ($80,948 × 1.023)	$84,797 ($82,809 × 1.024)
NOPAT		$10,820 ($77,652 × 13.93%)	$10,875 ($79,205 × 13.73%)	$10,803 ($80,948 × 13.35%)	$10,933 ($82,809 × 13.20%)	$10,955 ($84,797 × 12.92%)
NOA	$90,591	$94,729 ($79,205/0.836)	$99,087 ($80,948/0.817)	$102,910 ($82,809/0.805)	$105,116 ($84,797/0.807)	$107,638 ($86,832/0.807)

We can use these forecasts of *NOPAT* and *NOA* to obtain forecasts of free cash flow from operations (*FCF*), which is equal to *NOPAT* minus the change in *NOA*. Note that this is, in fact, the standard calculation seen in most valuation texts where change in net operating assets is broken into several main components, such as change in working capital (change in inventory, change in accounts receivable, and change in accounts payable), depreciation, and capital expenditure. The calculation of free cash flows would then be as shown in **Exhibit 10.5**.

EXHIBIT 10.5 PG Multiyear Forecasts of *FCF* ($ millions)

	2015	2016E	2017E	2018E	2019E	2020E
Sales.	$76,279	$77,652	$79,205	$ 80,948	$ 82,809	$ 84,797
NOPAT.		10,820	10,875	10,803	10,933	10,955
NOA	90,591	94,729	99,087	102,910	105,116	107,638
Δ*NOA*		4,138	4,358	3,823	2,206	2,523
FCF (*NOPAT*–Δ*NOA*).		6,682	6,517	6,980	8,727	8,432

While our analyses have provided forecasts of *FCF* from operations through 2020 (the fifth forecast period), the formula in Equation 10.6 requires expectations of all future cash flows (i.e., including years well beyond 2020); in other words, value arises from all future cash flows expected to be received. Obviously, trying to forecast an infinite stream is impossible whether we forecast *FCF* directly or we compute it indirectly from a stream of *NOPAT* and *NOA*. We, next, modify the DCF valuation model to include an estimate of the continuing value (i.e., the forecast of value at the end of the finite forecast horizon, which captures the value of expected payoffs in all periods beyond the horizon).

THE USE OF CONTINUING VALUES IN THE FREE CASH FLOW MODEL

When computing value using the free cash flow model, we partition the future into two separate pieces. The first is the finite-horizon period for which we have formed explicit forecasts (the first five years through 2020 in our PG example) and the second is the period beyond this horizon (the period from 2021 to infinity). Within the horizon, we have explicit forecasts. A continuing value estimate is made to capture expected payoffs beyond the finite forecast horizon.

Equation 10.6 can be rewritten to emphasize this split into two pieces as follows:

$$
\begin{aligned}
V_0 &= \frac{FCF_1}{(1+r_{WACC})} + \frac{FCF_2}{(1+r_{WACC})^2} + \frac{FCF_3}{(1+r_{WACC})^3} + \frac{FCF_4}{(1+r_{WACC})^4} + \frac{FCF_5}{(1+r_{WACC})^5} + \ldots \\
&= \sum_{t=1}^{\infty}\left(\frac{FCF_t}{(1+r_{WACC})^t}\right) \\
&= \sum_{t=1}^{5}\left(\frac{FCF_t}{(1+r_{WACC})^t}\right) + \sum_{t=6}^{\boxed{\infty}}\left(\frac{FCF_t}{(1+r_{WACC})^t}\right)
\end{aligned}
\qquad (10.7)
$$

In Equation 10.7, the term capturing the first five periods is referred to as being within the forecast horizon as it will be computed using forecasts of free cash flows for each year. The amount of effort expended to calculate these forecasts within the horizon can vary. Currently we are working with detailed "full information" forecasts where great levels of detail influenced the forecasts. Remember that analysts sometimes spend many hours/weeks working on the forecasts of sales growth alone!

The length of the forecast horizon will be determined by our ability and willingness to gather information for forecasting. We would like to explicitly forecast as far into the future as possible, but we have to recognize that the farther into the future we forecast the less confidence we have in our estimates. Consider estimates of your future earnings. Are you more confident in your estimate of your earnings next year or your estimate of your earnings 20 years from now? How far out into the future do you have to go before the estimates become mostly speculation? Similar issues are in play when we form forecasts for firms.

The second term of Equation 10.7 captures the continuing value beyond the explicit forecast period. To help show what is captured by the continuing value, we rewrite the second term of the equation as follows:

$$
\begin{aligned}
\sum_{t=6}^{\infty}\left(\frac{FCF_t}{(1+r_{WACC})^t}\right) &= \frac{1}{(1+r_{WACC})^5} \times \sum_{t=6}^{\infty}\left(\frac{FCF_t}{(1+r_{WACC})^{t-5}}\right) \\
&= \frac{1}{(1+r_{WACC})^5} \times \left(\frac{FCF_6}{(1+r_{WACC})} + \frac{FCF_7}{(1+r_{WACC})^2} + \frac{FCF_8}{(1+r_{WACC})^3} + \frac{FCF_9}{(1+r_{WACC})^4} + \ldots\right)
\end{aligned}
$$

Which, based on Equation 10.6, may be expressed as:

$$
\begin{aligned}
\sum_{t=6}^{\infty}\left(\frac{FCF_t}{(1+r_{WACC})^t}\right) &= \frac{1}{(1+r_{WACC})^5} \times \left(\frac{FCF_6}{(1+r_{WACC})} + \frac{FCF_7}{(1+r_{WACC})^2} + \frac{FCF_8}{(1+r_{WACC})^3} + \frac{FCF_9}{(1+r_{WACC})^4} + \ldots\right) \\
&= \frac{1}{(1+r_{WACC})^5} \times V_5
\end{aligned}
$$

That is, in the DCF model, the continuing value equals the expected intrinsic value at the horizon. This creates an interesting problem; in order to determine the value today we need an estimate of the value of the operating activities several periods in the future. We will return to this issue in future modules as we discuss alternative valuation methods.

To estimate the value at the horizon, an assumption that future payoffs will follow a predictable pattern beyond the horizon is, generally, made. That is, each subsequent payoff is related to the previous payoff in a systematic manner. Usually payoffs are assumed to be related such that they show growth at a constant rate. As a result, the following is assumed to hold:

$$
FCF_{t+1} = FCF_t \times (1+g)
$$

where g is the rate of growth in free cash flows from operations after the forecast horizon. Further, the subsequent payoff (FCF_{t+2}) will reflect two periods of growth such that:

$$
\begin{aligned}
FCF_{t+2} &= FCF_{t+1} \times (1+g) \\
&= \left[FCF_t \times (1+g)\right] \times (1+g) \\
&= FCF_t \times (1+g)^2
\end{aligned}
$$

USE OF NOMINAL (RISK-ADJUSTED) VS. REAL PAYOFFS

The value of a monetary unit commonly changes over time due to inflation or deflation. Payoffs and discount rates that include the effects of these price-level changes are referred to as nominal or nominal risk-adjusted. If the effects of price-level changes are not included, the payoffs and discount rates are referred to as real. So, should we use nominal or real amounts for valuation? Consider the following example where a nominal payoff of $200 million is expected in two years. Assume that there is 3 percent inflation per year expected and the nominal discount rate for this payoff is 10 percent, which implies a real discount rate of 6.8 percent (i.e., $\left[\frac{(1.00+0.10)}{(1.00+0.03)} - 1.00\right]$). We can estimate the value of the expected future payoff using either nominal or real amounts as follows:

Inputs:	Payoff	×	Discount Rate	=	Value Today
Nominal:	$200 million (nominal)	×	$1/1.10^2$ (nominal—includes inflation)	=	$165.3 million
Real:	$200 million/1.032 (real)	×	$1/1.068^2$ (real—excludes inflation)	=	$165.3 million

The computed value of the payoff is unaffected by whether nominal or real inputs are used in the calculation (remember that use of nominal payoffs requires use of nominal discount rates and use of real payoffs requires use of real discount rates). In practice, it is easier to use nominal inputs as analysts generally do not forecast real payoffs nor do they take the time to convert nominal rates to real rates. Accordingly, we use nominal inputs for valuations in this book.

FREE CASH FLOW VALUATION OF PROCTER & GAMBLE'S ENTERPRISE OPERATIONS

For the sake of valuation of PG using our detailed forecasts from Module 7, we will utilize a weighted average cost of capital for operations of 5.9 percent as computed in Module 9. In addition to the assumptions in **Exhibit 10.3**, we assume that the sales growth rate will stay at 2.4 percent in 2021 and then rise to 2.5 percent thereafter. We also assume that the rate of growth in free cash flow, essentially the rate of sales growth, beyond the explicit forecast horizon, will be 2.5 percent. The assumption that the rate of growth in free cash flows and the rate of growth in sales beyond the horizon are equal, reflects that, over time a firm's growth is driven by its ability to grow sales. If we apply our forecasts going forward. we can see that a steady state where free cash flows grow at the rate of growth in sales arises. The forecasts of free cash flows from operations (essentially **Exhibit 10.5** extended) are shown in **Exhibit 10.6**.

EXHIBIT 10.6 PG Multiyear Assumptions and Forecasts

	2016E	2017E	2018E	2019E	2020E	2021E	2022E	2023E
Assumptions:								
Sales growth	1.80%	2.00%	2.20%	2.30%	2.40%	2.40%	2.50%	2.50%
PM	13.93%	13.73%	13.35%	13.20%	12.92%	12.92%	12.92%	12.92%
ATO	0.858	0.836	0.817	0.805	0.807	0.807	0.807	0.807
Forecast (millions):								
Sales	$77,652	$79,205	$ 80,948	$ 82,809	$ 84,797	$ 86,832	$ 89,003	$ 91,228
NOPAT	10,820	10,875	10,803	10,933	10,955	11,218	11,498	11,786
NOA	94,729	99,087	102,910	105,116	107,638	110,329	113,088	115,915
Δ*NOA*	4,138	4,358	3,823	2,206	2,523	2,691	2,758	2,827
FCF (*NOPAT* − Δ*NOA*)	6,682	6,517	6,980	8,727	8,432	8,527	8,740	8,959

Here we can see that we achieve a steady state beginning in 2021 as all subsequent growth is at a constant rate of 2.5 percent. That is, we see steady/constant growth in *FCF* for 2022 and 2023 and it would continue in all subsequent years as well. For implementing Equation 10.8, we will use a period of six years as our explicit forecast horizon and then use a continuing value to capture periods seven and beyond. Using this information and our assumptions of the weighted average cost of capital for operations and the growth rate beyond the horizon in Equation 10.8, we calculate the value of PG's operations as follows:

$$\begin{aligned} FCF_t &\equiv NOPAT_t + NOA_{t-1} - NOA_t \\ &\equiv NOPAT_t - \boxed{\Delta NOA_t} \end{aligned} \tag{2.1}$$

Substituting for the cash flows (using Equation 2.1) into the cash flow valuation for the project in Equation 10.5, we see the following:

$$\begin{aligned} V_0 &= \frac{CF_1}{(1+r_P)} + \frac{CF_2}{(1+r_P)^2} + \frac{CF_3}{(1+r_P)^3} + \frac{CF_4}{(1+r_P)^4} \\ &= \frac{NOPAT_1 + NOA_0 - NOA_1}{(1+r_P)} + \frac{NOPAT_2 + NOA_1 - NOA_2}{(1+r_P)^2} \\ &+ \frac{NOPAT_3 + NOA_2 - NOA_3}{(1+r_P)^3} + \frac{NOPAT_4 + NOA_3 - NOA_4}{(1+r_P)^4} \\ &= \frac{50.00 + 1{,}000.00 - 800.00}{(1+0.10)} + \frac{150.00 + 800.00 - 600.00}{(1+0.10)^2} \\ &+ \frac{120.00 + 600.00 - 400.00}{(1+0.10)^3} + \frac{80.00 + 400.00 - 0.00}{(1+0.10)^4} \\ &= \frac{250.00}{1.10} + \frac{350.00}{1.21} + \frac{320.00}{1.331} + \frac{480.00}{1.4641} \\ &= 227.27 + 289.26 + 240.42 + 327.85 \\ &= 1{,}084.80 \end{aligned}$$

While continuing to find a value of $1,084.80 when computing value using accounting earnings and book values probably appears obvious given the substitution we made, many trained in valuation would not find the fact that we have valued the operations using *only* accounting numbers obvious at all. We now examine the implications for substituting Equation 2.1 into Equation 10.5 as we develop the first of two accounting-based valuation models that are mathematically equivalent to DCF valuation. Later, we will discuss why you may wish to know several valuation models and what factors would determine which model may be used in a particular valuation setting.

DERIVATION OF THE RESIDUAL OPERATING INCOME VALUATION MODEL

Now that we have seen that we can use forecasted net operating profit after tax (*NOPAT*) and net operating assets (*NOA*) to value the project, we introduce and derive the first valuation model based on these accounting numbers. The derivation is simple and straightforward if we are prepared to pause so that we can carefully understand two unequivocal and indisputable steps. We will first see these steps now in the derivation of the residual operating income valuation model and then, again, in the derivation of the abnormal operating income growth valuation model.

The derivation of the residual operating income valuation model begins with the free cash flow valuation model, which we saw in Module 10:

$$\begin{aligned} V_0 &= \frac{FCF_1}{(1+r_{WACC})} + \frac{FCF_2}{(1+r_{WACC})^2} + \frac{FCF_3}{(1+r_{WACC})^3} + \frac{FCF_4}{(1+r_{WACC})^4} + \dots \\ &= \sum_{t=1}^{\infty} \left(\frac{FCF_t}{(1+r_{WACC})^t} \right) \end{aligned} \tag{10.6}$$

many of the accusations put forth by proponents of cash flow-based valuation are totally invalid. More discussion of this fact will arise going forward.

Having seen that the residual operating income valuation model produced the same value estimate for our project as the free cash flow model, we proceed to the valuation of PG's operations based on our detailed forecasts. Here we will see the implementation of the residual operating income valuation model for a going concern. This situation will include the use of a continuing value similar to that seen in the free cash flow model.

DETERMINING VALUE BASED ON THE FORECASTS OF OPERATIONS FOR PROCTER & GAMBLE

We will again employ the following assumptions shown in **Exhibit 11.5** for the foreseeable future used to form forecasts of operations for PG after review of its 2015 financial statements:

EXHIBIT 11.5 PG Multiyear Forecasts of Sales Growth, *PM*, and *ATO*

	2016E	2017E	2018E	2019E	2020E
Sales growth	1.80%	2.00%	2.20%	2.30%	2.40%
Operating *PM*	13.93%	13.73%	13.35%	13.20%	12.92%
Operating *ATO*	0.858	0.836	0.817	0.805	0.807

As in Modules 7 and 10, these assumptions lead to the forecasted numbers shown in **Exhibit 11.6** for 2016 through 2020.

EXHIBIT 11.6 PG Multiyear Forecasts of Sales, *NOPAT*, and *NOA* ($ millions)

	2015	2016E	2017E	2018E	2019E	2020E
Sales	$76,279	$77,652 ($76,279 × 1.018)	$79,205 ($77,652 × 1.020)	$80,948 ($79,205 × 1.022)	$82,809 ($80,948 × 1.023)	$84,797 ($82,809 × 1.024)
NOPAT		$10,820 ($77,652 × 13.93%)	$10,875 ($79,205 × 13.73%)	$10,803 ($80,948 × 13.35%)	$10,933 ($82,809 × 13.20%)	$10,955 ($84,797 × 12.92%)
NOA	$90,591	$94,729 ($79,205/0.836)	$99,087 ($80,948/0.817)	$102,910 ($82,809/0.805)	$105,116 ($84,797/0.807)	$107,638 ($86,832/0.807)

While we could use these forecasts of *NOPAT* and *NOA* to obtain forecasts of free cash flow from operations (*FCF*) as was done in the previous module, this step is not necessary. Based on Equation 11.2, it is clear that value can be computed directly from the accounting information. While we have forecasts through 2020 (the fifth forecast period), the formula in Equation 11.2 requires the series of all future *NOPAT* and *NOA*. Again, it is obviously impossible to forecast an infinite stream of *NOPAT* and *NOA*. Therefore, we consider the roles of forecasts within an explicit forecast horizon and a forecasted continuing value that captures all periods occurring after the horizon.

THE USE OF CONTINUING VALUES IN THE RESIDUAL OPERATING INCOME VALUATION MODEL

In computing value using the residual operating income valuation model, we again think of the future as containing two separate pieces that are added together just as was done in the free cash flow model. The first is a finite-horizon period for which we have formed explicit forecasts of *ROPI* (the first six years through 2021 in our PG example) and the second is the period beyond this horizon (the period from 2022 to infinity). Within the horizon we have explicit forecasts where we directly calculate expectations of the forecasts. However, to capture any *ROPI* occurring after the horizon, a continuing value, which captures the expectations beyond the horizon, must be calculated. As we will show, this second piece will capture a different proportion of value than was captured in the free cash flow model.

Equation 11.2 can be rewritten to emphasize this split into two pieces as follows:

$$\begin{aligned} V_0 &= NOA_0 + \sum_{t=1}^{\infty}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right) \\ &= NOA_0 + \sum_{t=1}^{6}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right) + \sum_{t=7}^{\infty}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right) \\ &= NOA_0 + \sum_{t=1}^{6}\left(\frac{ROPI_t}{(1+r_{WACC})^t}\right) + \sum_{t=7}^{\infty}\left(\frac{ROPI_t}{(1+r_{WACC})^t}\right) \end{aligned} \tag{11.3}$$

In Equation 11.3, the term capturing the first six periods is referred to as being within the forecast horizon as it will be computed using forecasts of *NOPAT* and *NOA* for each of the years within the forecast horizon.

The second term of Equation 11.3 captures the continuing value beyond the explicit forecast period. To help show what is captured by the continuing value when using the residual operating income valuation model, we rewrite Equation 11.2 as follows:

$$V_0 = NOA_0 + \sum_{t=1}^{\infty}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right)$$

$$V_0 - NOA_0 = \sum_{t=1}^{\infty}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right)$$

or more generally at any point in time T:

$$V_T - NOA_T = \sum_{t=1}^{\infty}\left(\frac{NOPAT_{T+t} - r_{WACC} \times NOA_{T+t-1}}{(1+r_{WACC})^t}\right)$$

From this formula, the difference between value and the amount recorded as *NOA* at any point in time T is the discounted present value of all future $ROPI_t$ beyond time T. This means that Equation 11.3 can be restated as:

all references to r_{Ent} should be r_{WACC}

$$\begin{aligned} V_0 &= NOA_0 + \sum_{t=1}^{6}\left(\frac{NOPAT_t - \cancel{r_{Ent}} \times NOA_{t-1}}{(1+\cancel{r_{Ent}})^t}\right) + \sum_{t=7}^{\infty}\left(\frac{NOPAT_t - \cancel{r_{Ent}} \times NOA_{t-1}}{(1+\cancel{r_{Ent}})^t}\right) \\ &= NOA_0 + \sum_{t=1}^{6}\left(\frac{NOPAT_t - \cancel{r_{Ent}} \times NOA_{t-1}}{(1+\cancel{r_{Ent}})^t}\right) + \frac{1}{(1+\cancel{r_{Ent}})^6}\sum_{t=7}^{\infty}\left(\frac{NOPAT_t - \cancel{r_{Ent}} \times NOA_{t-1}}{(1+\cancel{r_{Ent}})^{t-6}}\right) \\ &= NOA_0 + \sum_{t=1}^{6}\left(\frac{NOPAT_t - \cancel{r_{Ent}} \times NOA_{t-1}}{(1+\cancel{r_{Ent}})^t}\right) + \frac{1}{(1+\cancel{r_{Ent}})^6}(V_6 - NOA_6) \end{aligned}$$

That is, in the residual operating income valuation model, the continuing value equals the expected difference between intrinsic value and book value of the operations at the horizon. This amount is sometimes referred to as the premium over book value. In the free cash flow model, a problem was that we needed to know the value, several periods in the future to determine the value today. Here we only need to capture the excess of intrinsic value over book value which could even be equal to zero.

To estimate the continuing value at the forecast horizon, we will most often make an assumption that future residual operating income, $NOPAT_t - r_{WACC} \times NOA_{t-1}$, will follow a predictable pattern beyond the horizon. That is, each subsequent amount is related to the previous amount in a systematic manner. Residual operating income can be assumed to show growth, be constant or fade over time. As a result, we will assume the following holds:

$$NOPAT_{t+1} - r_{WACC} \times NOA_t = (NOPAT_t - r_{WACC} \times NOA_{t-1}) \times (1+g)$$

$$ROPI_{t+1} = ROPI_t \times (1+g)$$

where g is the growth rate in residual operating income after the forecast horizon. Further, the subsequent amount of residual operating income ($NOPAT_{t+2} - r_{WACC} \times NOA_{t+1}$) will reflect two periods of growth such that:

$$\begin{aligned} NOPAT_{t+2} - r_{WACC} \times NOA_{t+1} &= (NOPAT_{t+1} - r_{WACC} \times NOA_t) \times (1+g) \\ &= \left[(NOPAT_t - r_{WACC} \times NOA_{t-1}) \times (1+g)\right] \times (1+g) \\ &= (NOPAT_t - r_{WACC} \times NOA_{t-1}) \times (1+g)^2 \end{aligned}$$

$$ROPI_{t+2} = ROPI_t \times (1+g)^2$$

The pattern that emerges allows us to continue the refining of our understanding of the second term of our valuation equation as follows:

$$\sum_{t=7}^{\infty}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right) = \frac{1}{(1+\cancel{r_{Ent}})^6} \times \left(\frac{NOPAT_7 - r_{WACC} \times NOA_6}{(1+r_{WACC})} + \frac{NOPAT_8 - r_{WACC} \times NOA_7}{(1+r_{WACC})^2} + \frac{NOPAT_8 - r_{WACC} \times NOA_8}{(1+r_{WACC})^3}\right)$$

r_{Ent} should be r_{WACC}

$$= \frac{1}{(1+r_{WACC})^6} \times \left(\frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)}{(1+r_{WACC})} + \frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)^2}{(1+r_{WACC})^2} + \frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)^3}{(1+r_{WACC})^3} + \ldots\right)$$

$$= \frac{1}{(1+r_{WACC})^6} \times \left(\frac{ROPI_6 \times (1+g)}{(1+r_{WACC})} + \frac{ROPI_6 \times (1+g)^2}{(1+r_{WACC})^2} + \frac{ROPI_6 \times (1+g)^3}{(1+r_{WACC})^3} + \ldots\right)$$

The last portion of the above equation shows a series that will continue to infinity in a very straightforward manner. Each subsequent term has the numerator multiplied by $(1 + g)$ and the denominator multiplied by $(1 + r_{WACC})$. Provided that the discount rate is greater than the growth rate ($r_{WACC} > g$) this series can be rewritten as follows:

$$\sum_{t=7}^{\infty}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right) = \frac{1}{(1+\cancel{r_{Ent}})^6} \times \left(\frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)}{(1+r_{WACC})} + \frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)^2}{(1+r_{WACC})^2} + \frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)^3}{(1+r_{WACC})^3} + \ldots\right)$$

all references to r_{Ent} should be r_{WACC}

$$= \frac{1}{(1+\cancel{r_{Ent}})^6} \times \left(\frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)}{r_{WACC} - g}\right)$$

$$= \frac{1}{(1+\cancel{r_{Ent}})^6} \times \left(\frac{ROPI_6 \times (1+g)}{r_{WACC} - g}\right)$$

Combining this with Equation 11.3 yields:

$$
\begin{aligned}
V_0 &= NOA_0 + \frac{NOPAT_1 - r_{WACC} \times NOA_0}{(1+r_{WACC})} + \frac{NOPAT_2 - r_{WACC} \times NOA_1}{(1+r_{WACC})^2} \\
&\quad + \frac{NOPAT_3 - r_{WACC} \times NOA_2}{(1+r_{WACC})^3} + \ldots \\
&= NOA_0 + \sum_{t=1}^{\infty}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right) \\
&= NOA_0 + \sum_{t=1}^{6}\left(\frac{NOPAT_t - r_{WACC} \times NOA_{t-1}}{(1+r_{WACC})^t}\right) \\
&\quad + \frac{1}{(1+r_{WACC})^6} \times \left(\frac{(NOPAT_6 - r_{WACC} \times NOA_5) \times (1+g)}{r_{WACC} - g}\right) \\
&= NOA_0 + \sum_{t=1}^{6}\left(\frac{ROPI_t}{(1+r_{WACC})^t}\right) + \frac{1}{(1+r_{WACC})^6} \times \left(\frac{ROPI_6 \cdot (1+g)}{r_{WACC} - g}\right)
\end{aligned}
\tag{11.4}
$$

Note that the growth rate in Equation 11.4 may not be the same as the growth rate in Equation 10.8 as one captures future growth in residual operating income and the other captures future growth in free cash flows from operations. In our examples in this module, the growth rate for residual operating income is the same as the growth rate for free cash flow. We will explain the reason for this in Module 13. As a reminder, Equation 11.4 holds when the growth rate from year 6 to 7 and all subsequent growth is at g.

RESIDUAL OPERATING INCOME VALUATION OF PROCTER & GAMBLE'S OPERATIONS

When valuing PG's operations using our forecasts formed in Module 7, we continue to assume a weighted average cost of capital for operations of 5.9 percent as estimated in Module 9. In addition to the assumptions in **Exhibit 11.5**, we assume (as we did in our DCF example) that the sales growth rate stays at 2.4 percent in 2021 and then rises to 2.5 percent thereafter. We also assume that the growth rate in residual operating income beyond the explicit forecast horizon will be equal to the growth rate in sales of 2.5 percent. The assumption, that the growth rate in sales and the growth rate in residual operating income will converge, captures the notion that, over time, a firm's growth is driven by its ability to grow sales. If we apply our forecasts going forward, the forecasts of *NOPAT* and *NOA* (essentially **Exhibit 11.6** extended) are as shown in **Exhibit 11.7**.

EXHIBIT 11.7 PG Multiyear Assumptions and Forecasts

	2016E	2017E	2018E	2019E	2020E	2021E	2022E
Assumptions:							
Sales growth	1.80%	2.00%	2.20%	2.30%	2.35%	2.35%	2.40%
Operating *PM*	13.93%	13.73%	13.35%	13.20%	12.92%	12.92%	12.92%
Operating *ATO*	0.858	0.836	0.817	0.805	0.807	0.807	0.807
Forecast (millions):							
Sales	$77,652	$79,205	$ 80,948	$ 82,809	$ 84,797	$ 86,832	$ 89,003
NOPAT	10,820	10,875	10,803	10,933	10,955	11,218	11,498
NOA	94,729	99,087	102,910	$105,116	107,638	110,329	113,088

MODULE OUTLINE

Valuation using the Abnormal Operating Income Growth Model

Abnormal Operating Income Growth Valuation
- Derivation from Cash Flows Model
- Role of Cum-Free-Cash-Flow Earnings
- Valuing a Finite-Life Project

Accounting Choices
- Effects of Alternative Depreciation Methods

Valuation of Operations
- Multiyear Forecasts
- Continuing Values
- Valuing Procter & Gamble's Operations

EXHIBIT 12.1 Anticipated Cash Flows of Finite-Life Project

Purchase price		$1,000.00
Payoffs		
Year 1 cash flow (CF_1)		250.00
Year 2 cash flow (CF_2)		350.00
Year 3 cash flow (CF_3)		320.00
Year 4 cash flows (CF_4)		
Proceeds from sales	$280.00	
Proceeds from disposition	200.00	
		480.00
Required rate of return = r_P		10%

As a reminder, Equation 10.5 showed that we can value a project using cash flows as follows:

$$V_0 = \frac{CF_1}{(1+r_P)} + \frac{CF_2}{(1+r_P)^2} + \frac{CF_3}{(1+r_P)^3} + \frac{CF_4}{(1+r_P)^4} = \sum_{t=1}^{4}\left(\frac{CF_t}{(1+r_P)^t}\right) \tag{10.5}$$

When the expected cash flows for the project were entered into the above formula, we computed the following present value:

$$\begin{aligned} V_0 &= \frac{CF_1}{(1+r_P)} + \frac{CF_2}{(1+r_P)^2} + \frac{CF_3}{(1+r_P)^3} + \frac{CF_4}{(1+r_P)^4} \\ &= \frac{250.00}{(1+0.10)^1} + \frac{350.00}{(1+0.10)^2} + \frac{320.00}{(1+0.10)^3} + \frac{480.00}{(1+0.10)^4} \\ &= \frac{250.00}{1.1} + \frac{350.00}{1.21} + \frac{320.00}{1.331} + \frac{480.00}{1.4641} \\ &= 227.27 + 289.26 + 240.42 + 327.85 \\ &= 1{,}084.80 \end{aligned}$$

In Module 11, we moved from cash flow-based valuation to accounting-based valuation with the aid of the following equation arrived at in Module 2:

$$\begin{aligned} FCF_t &\equiv NOPAT_t + NOA_{t-1} - NOA_t \\ &\equiv NOPAT_t - \boxed{\Delta NOA_t} \end{aligned} \tag{2.1}$$

Also in Module 11, we utilized a zero-sum equation that anchored on *NOA*. This allowed for the derivation of the residual operating income valuation model. In this module, we again utilize a zero-sum equation but instead of anchoring on *NOA*, we will anchor on capitalized expected net operating profit after tax ($NOPAT/r_{WACC}$).

DERIVATION OF THE ABNORMAL OPERATING INCOME GROWTH VALUATION MODEL

The derivation of the abnormal operating income growth valuation model follows the same steps as the derivation of the residual operating income model. The derivation again is simple and straightforward if we are prepared to pause so that we can carefully understand the two unequivocal and indisputable steps. Again, the derivation begins with the discounted free cash flow valuation model:

$$\begin{aligned} V_0 &= \frac{FCF_1}{(1+r_{WACC})} + \frac{FCF_2}{(1+r_{WACC})^2} + \frac{FCF_3}{(1+r_{WACC})^3} + \frac{FCF_4}{(1+r_{WACC})^4} + \ldots \\ &= \sum_{t=1}^{\infty}\left(\frac{FCF_t}{(1+r_{WACC})^t}\right) \end{aligned} \tag{10.6}$$

The next step is introduction of the valuation anchor to be used in this model; that is, capitalized next period net operating profit after tax $\left({}^{NOPAT_1}/_{r_{WACC}}\right)$. Similar to the derivation of the residual operating income valuation model, we introduce this starting point by recognizing the simple mathematical fact that, if we add and, at the same time, subtract the present value of each future year's capitalized *NOPAT* in the right hand side of Equation 10.6, we will not affect the mathematical integrity of the equation. Specifically, we introduce the following zero-sum equality:

$$\begin{aligned} 0 = {}& {}^{NOPAT_1}/_{r_{WACC}} + \frac{{}^{NOPAT_2}/_{r_{WACC}} - (1+r_{WACC}) \times {}^{NOPAT_1}/_{\cancel{r_{Ent}}}}{(1+r_{WACC})} \\ &+ \frac{{}^{NOPAT_3}/_{r_{WACC}} - (1+r_{WACC}) \times {}^{NOPAT_2}/_{\cancel{r_{Ent}}}}{(1+r_{WACC})^2} + \ldots\ldots \end{aligned} \tag{12.1}$$

r_{Ent} should be r_{WACC}

The fact that we have simply added and subtracted the present value of each future period's capitalized *NOPAT* is more evident if we rearrange Equation 12.1 as follows:

$$\begin{aligned} 0 = {}& {}^{NOPAT_1}/_{r_{WACC}} - {}^{NOPAT_1}/_{r_{WACC}} + \frac{{}^{NOPAT_2}/_{r_{WACC}}}{(1+r_{WACC})} - \frac{{}^{NOPAT_2}/_{r_{WACC}}}{(1+r_{WACC})} \\ &+ \frac{{}^{NOPAT_3}/_{r_{WACC}}}{(1+r_{WACC})^2} - \frac{{}^{NOPAT_3}/_{r_{WACC}}}{(1+r_{WACC})^2} + \ldots\ldots \end{aligned}$$

The next step is to simply add the left hand sides of Equations 10.6 and 12.1 and to add the right hand sides to obtain:

$$V_0 = {}^{NOPAT_1}/_{r_{WACC}} + \sum_{t=1}^{\infty}\left(\frac{{}^{NOPAT_{t+1}}/_{r_{WACC}} + FCF_t - (1+r_{WACC}) \times {}^{NOPAT_t}/_{r_{WACC}}}{(1+r_{WACC})^t}\right)$$

the expected earnings we would have observed

the account ($30.30) plus the amount earned outside the account on the money distributed in the prior period ($0.60). This amount is the same as ~~if the expected earnings~~ had no distribution occurred ($30.90 = 3% × $1,030.00). In other words, cum-free-cash-flow earnings are not affected by the amount distributed in prior period while ex-free-cash-flow earnings (that is, *NOPAT*) are affected.

When computing cum-free-cash-flow earnings, we assume the distribution received was invested in an alternative with equivalent risk. Thus, a distribution from a savings account would be reinvested in an alternative with low risk, a distribution from the project would be reinvested in an alternative with a rate of return equal to r_P, and a distribution from the operations would be reinvested in an alternative with the rate of return equal to r_{WACC}, the weighted average cost of capital for operations.[2]

Now let us proceed to apply the abnormal operating income growth valuation model to the valuation of the finite-life project.

PROJECT VALUATION USING THE ABNORMAL OPERATING INCOME GROWTH MODEL

The four-period abnormal operating income growth valuation model based on Equation 12.3 is:

$$V_0 = \frac{1}{r_P} \times \left[NOPAT_1 + \frac{aoig_2}{(1+r_P)} + \frac{aoig_3}{(1+r_P)^2} + \frac{aoig_4}{(1+r_P)^3} + \frac{aoig_5}{(1+r_P)^4} \right]$$

where r_P again is the required discount rate for the project.[3] To observe the computation of *aoig*, we expand the formula as follows:

$$V_0 = \frac{1}{r_P} \times \left[\begin{array}{l} NOPAT_1 + \dfrac{NOPAT_2 + r_P \times FCF_1 - (1+r_P) \times NOPAT_1}{(1+r_P)} \\ + \dfrac{NOPAT_3 + r_P \times FCF_2 - (1+r_P) \times NOPAT_2}{(1+r_P)^2} \\ + \dfrac{NOPAT_4 + r_P \times FCF_3 - (1+r_P) \times NOPAT_3}{(1+r_P)^3} \\ + \dfrac{NOPAT_5 + r_P \times FCF_4 - (1+r_P) \times NOPAT_4}{(1+r_P)^4} \end{array} \right]$$

Recall the following accounting information about the project from Module 10 as shown in **Exhibit 12.2**.

EXHIBIT 12.2 Accounting Income and Distributions for Finite-Life Project

Time period	0	1	2	3	4
Net income ($NOPAT_t$)		50.00	150.00	120.00	80.00
Distributions (FCF_t)	(1,000.00)	250.00	350.00	320.00	480.00

[2] The idea here is that if your chosen investment is a low-risk savings account, this is the risk level at which you are willing to make other investments. Also, note that, if, instead of choosing to re-invest the withdrawn funds from your savings account, you choose to spend these funds on, say, theater tickets, the opportunity cost of this spending is, in this example, 3 percent and you have foregone earnings in the next period of $0.60. The notion that an alternative investment with equal risk is available is standard in finance.

[3] Note that $aoig_1$ does not enter the computation. We notate the *aoig* in such a way that the first term (the *NOPAT*) has the same subscript as the *aoig*.

EXHIBIT 12.5 PG Multiyear Forecasts of Sales Growth, *PM*, and *ATO*

	2016E	2017E	2018E	2019E	2020E
Sales growth	1.80%	2.00%	2.20%	2.30%	2.40%
Operating *PM*	13.93%	13.73%	13.35%	13.20%	12.92%
Operating *ATO*	0.858	0.836	0.817	0.805	0.807

As in Modules 7, 10, and 11, these assumptions lead to the forecasted numbers shown in **Exhibit 12.6** for 2016 through 2020.

EXHIBIT 12.6 PG Multiyear Forecasts of Sales, *NOPAT*, and *NOA* ($ millions)

	2015	2016E	2017E	2018E	2019E	2020E
Sales	$76,279	$77,652 ($76,279 × 1.018)	$79,205 ($77,652 × 1.020)	$80,948 ($79,205 × 1.022)	$82,809 ($80,948 × 1.023)	$84,797 ($82,809 × 1.024)
NOPAT		$10,820 ($77,652 × 13.93%)	$10,875 ($79,205 × 13.73%)	$10,803 ($80,948 × 13.35%)	$10,933 ($82,809 × 13.20%)	$10,955 ($84,797 × 12.92%)
NOA	$90,591	$94,729 ($79,205/0.836)	$99,087 ($80,948/0.817)	$102,910 ($82,809/0.805)	$105,116 ($84,797/0.807)	$107,638 ($86,832/0.807)

We can use these forecasts of *NOPAT* and *NOA* to obtain the forecasts of free cash flow (*FCF*), which is equal to *NOPAT* minus the change in *NOA*. The calculation of free cash flows from operations utilized in the abnormal operating income growth model and the *aoig* would then be as shown in **Exhibit 12.7**.

EXHIBIT 12.7 PG Multiyear Forecasts of *aoig* ($ millions)

	2015	2016E	2017E	2018E	2019E	2020E
Sales	$76,279	$77,652	$79,205	$ 80,948	$ 82,809	$ 84,797
NOPAT		10,820	10,875	10,803	10,933	10,955
NOA	90,591	94,729	99,087	102,910	105,116	107,638
Δ*NOA*		4,138	4,358	3,823	2,206	2,523
FCF (*NOPAT* − Δ*NOA*)		6,682	6,517	6,980	8,727	8,432
aoig			(189)	(328)	(96)	(108)

Thus we have forecasts of *NOPAT* and *FCF* to allow us to utilize the abnormal operating income growth model for PG. While we have forecasts through 2020 (the fifth forecast period), the formula in Equation 12.3 requires the series of all future *NOPAT* and *FCF*. Again, it is obviously impossible to forecast an infinite stream of *NOPAT* and *FCF*. Therefore, we consider the roles of forecasts within an explicit forecast horizon and a forecasted continuing value that captures all periods occurring after the horizon.

THE USE OF CONTINUING VALUES IN THE ABNORMAL OPERATING INCOME GROWTH VALUATION MODEL

In computing value using the abnormal operating income growth valuation model, we need to think of the future as containing two separate pieces that are added together just as was done in the free cash flow model and residual operating income model. The first is a finite-horizon period for which we will form explicit forecasts of *aoig* (the first six realizations of *aoig* through 2022 in our PG example) and the second is the period beyond this horizon (the period from 2023 to infinity).[4] Within the horizon we have explicit forecasts where we directly calculate expectations of the forecasts. However, to capture any *aoig* occurring after the horizon, a continuing value must be calculated that captures the expectations for that period. As we will show, this second piece represents something different from that captured in the free cash flow model or in the residual operating income model.

[4] Note that when using the abnormal operating income growth model, *aoig* reaches steady-state a year later than either *FCF* or *ROPI*. This is because the calculation of cum-dividend earnings is based on lagged *FCF*, which is in turn, based on (further) lagged *NOA*, so that it takes one more year for *aoig* to be growing at the steady state growth rate.

The pattern that emerges allows us to continue the refining of our understanding of the second term of our valuation equation as follows:

$\boxed{\text{all references to } r_{Ent} \text{ should be } r_{WACC}}$

$$\sum_{t=7}^{\infty}\left(\frac{NOPAT_{t+1}+r_{WACC}\times FCF_t-(1+r_{WACC})\times NOPAT_t}{(1+r_{WACC})^t}\right)=\frac{1}{(1+r_{WACC})^6}\times\left(\begin{array}{l}\dfrac{NOPAT_8+\cancel{r_{Ent}}\times FCF_7-(1+r_{WACC})\times NOPAT_7}{(1+r_{WACC})}\\[2ex] +\dfrac{NOPAT_9+r_{WACC}\times FCF_8-(1+r_{WACC})\times NOPAT_8}{(1+r_{WACC})^2}\\[2ex] +\dfrac{NOPAT_{10}+\cancel{r_{Ent}}\times FCF_9-(1+r_{WACC})\times NOPAT_9}{(1+r_{WACC})^3}+\ldots\end{array}\right)$$

$$=\frac{1}{(1+r_{WACC})^6}\times\left(\begin{array}{l}\dfrac{[NOPAT_7+\cancel{r_{Ent}}\times FCF_6-(1+r_{WACC})\times NOPAT_6]\times(1+g)}{(1+r_{WACC})}\\[2ex] +\dfrac{[NOPAT_7+\cancel{r_{Ent}}\times FCF_6-(1+r_{WACC})\times NOPAT_6]\times(1+g)^2}{(1+r_{WACC})^2}\\[2ex] +\dfrac{[NOPAT_7+\cancel{r_{Ent}}\times FCF_6-(1+r_{WACC})\times NOPAT_6]\times(1+g)^3}{(1+r_{WACC})^3}\end{array}\right)$$

$$=\frac{1}{(1+r_{WACC})^6}\times\left(\frac{aoig_7\times(1+g)}{(1+r_{WACC})}+\frac{aoig_7\times(1+g)^2}{(1+r_{WACC})^2}+\frac{aoig_7\times(1+g)^3}{(1+r_{WACC})^3}+\ldots\right)$$

The last portion of the above equation shows a series that will continue to infinity in a very straightforward manner. Each subsequent term has the numerator multiplied by $(1 + g)$ and the denominator multiplied by $(1 + r_{WACC})$. Provided that the discount rate is greater than growth rate ($r_{WACC} > g$) this series can be rewritten as follows:

$\boxed{\text{all references to } r_{Ent} \text{ should be } r_{WACC}}$

$$\sum_{t=7}^{\infty}\left(\frac{NOPAT_{t+1}+r_{WACC}\times FCF_t-(1+r_{WACC})\times NOPAT_t}{(1+r_{WACC})^t}\right)=\frac{1}{(1+\cancel{r_{Ent}})^6}\times\left(\begin{array}{l}\dfrac{[NOPAT_7+\cancel{r_{Ent}}\times FCF_6-(1+r_{WACC})\times NOPAT_6]\times(1+g)}{(1+r_{WACC})}\\[2ex] +\dfrac{[NOPAT_7+\cancel{r_{Ent}}\times FCF_6-(1+r_{WACC})\cdot NOPAT_6]\times(1+g)^2}{(1+r_{WACC})^2}\\[2ex] +\dfrac{[NOPAT_7+\cancel{r_{Ent}}\times FCF_6-(1+r_{WACC})\times NOPAT_6]\cdot(1+g)^3}{(1+r_{WACC})^3}+\ldots\end{array}\right)$$

$$=\frac{1}{(1+\cancel{r_{Ent}})^6}\times\frac{[NOPAT_7+\cancel{r_{Ent}}\times FCF_6-(1+r_{WACC})\times NOPAT_6]\times(1+g)}{r_{WACC}-g}$$

$$=\frac{1}{(1+\cancel{r_{Ent}})^6}\times\frac{aoig_7\times(1+g)}{\cancel{r_{Ent}}-g}$$

EXHIBIT 12.8 PG Multiyear Assumptions and Forecasts

	2016E	2017E	2018E	2019E	2020E	2021E	2022E	2023E
Assumptions:								
Sales growth	1.80%	2.00%	2.20%	2.30%	2.40%	2.40%	2.50%	2.50%
Operating *PM*	13.93%	13.73%	13.35%	13.20%	12.92%	12.92%	12.92%	12.92%
Operating *ATO*	0.858	0.836	0.817	0.805	0.807	0.807	0.807	0.807
Forecast (millions):								
Sales.	$77,652	$79,205	$ 80,948	$ 82,809	$ 84,797	$ 86,832	$ 89,003	$ 91,228
NOPAT	10,820	10,875	10,803	10,933	10,955	11,218	11,498	11,786
NOA	94,729	99,087	102,910	105,116	107,638	110,329	113,088	115,915
Δ*NOA*	4,138	4,358	3,823	2,206	2,523	2,691	2,758	2,827
FCF (*NOPAT* − Δ*NOA*) . . .	6,682	6,517	6,980	8,727	8,432	8,527	8,740	8,959
aoig		(189)	(328)	(96)	(108)	114	122	125

EXHIBIT 12.9 Abnormal Operating Income Growth Model

Required rate of return (r_{WACC}) 5.90%

Time period	2015	2016	2017	2018	2019	2020	2021	2022
Forecasted sales	$ 76,279	$77,652	$79,205	$ 80,948	$ 82,809	$ 84,797	$ 86,832	$ 89,003
NOPAT.		10,820	10,875	10,803	10,933	10,955	11,218	11,498
NOA	90,591	94,729	99,087	102,910	105,116	107,638	110,329	113,088
Δ*NOA*.		4,138	4,358	3,823	2,206	2,523	2,691	2,758
FCF (*NOPAT* − Δ*NOA*)		6,682	6,517	6,980	8,727	8,432	8,527	8,740
$NOPAT_t$		10,820	10,875	10,803	10,933	10,955	11,218	11,498
$aoig_t$			−189	−328	−96	−108	114	122
Growth in *aoig*				73.41%	−70.77%	12.66%	−205.46%	6.67%
Discount factor $(1 + r_{WACC})^t$			1.0590	1.1215	1.1876	1.2577	1.3319	1.4105
Present value of $aoig_t$.			−179	−293	−81	−86	86	86
Sum of present value of $aoig_t$.		(467)						
Continuing value								
$aoig_{2023} = aoig_{2022} * (1 + 0.025)$ . . .								125
$aoig_{2023}/(0.059 - 0.025)$								3,668
Present value of continuing value. . . .		2,601						
Total to be capitalized.		12,954						
Operating value	219,559							

That is, the estimated intrinsic value of the enterprise operations of PG using the abnormal operating income growth valuation model is $219,559. Despite using accounting information to estimate value, we arrived at the same result as using the highly-touted free cash flow approach. Again, we emphasize that the fact that we used accounting numbers, which are subject to discretion in choice of accounting methods, did not alter the estimate of value because our assumptions are consistent across the models.

In this AOIGM valuation, the portion of value of the operations captured within the finite forecast horizon is over 79.9 percent [$175,480/$219,559]! Since almost 80 percent of the value is coming from the forecasts for the next four years we would potentially have higher confidence in this forecast. This is because we have a higher confidence in the near term forecasts. Clearly the anchoring of the value on *NOPAT* has greatly reduced the portion of the value estimate coming from extended forecasts; specifically, our reliance on the continuing value estimate is minimized.

SUMMARY

This module added another accounting-based valuation model to our arsenal. It also continued to demonstrate that accounting information can be used in lieu of free cash flows to value enterprise operations. Both the abnormal operating income growth model and the residual operating income model are driven by free cash flow being equal to net operating profit after taxes (*NOPAT*) adjusted for the change in net operating assets (*NOA*). They use different accounting-based anchors in the valuation.

EXHIBIT 13.3 Breakdown of PG Value Estimate based on Horizon

	Free Cash Flow Model		Residual Operating Income Model		Abnormal Operating Income Growth Model	
	Amount	Percent	Amount	Percent	Amount	Percent
No forecast necessary	$ 0	0.0%	$ 90,591	41.3%	$ 0	0.0%
Based on forecasts within horizon	37,313	17.0%	24,942	11.4%	175,480	79.9%
Based on continuing value	182,246	83.0%	104,026	47.4%	44,079	20.1%
Total estimated value	$219,559	100.0%	$219,559	100.0%	$219,559	100.0%

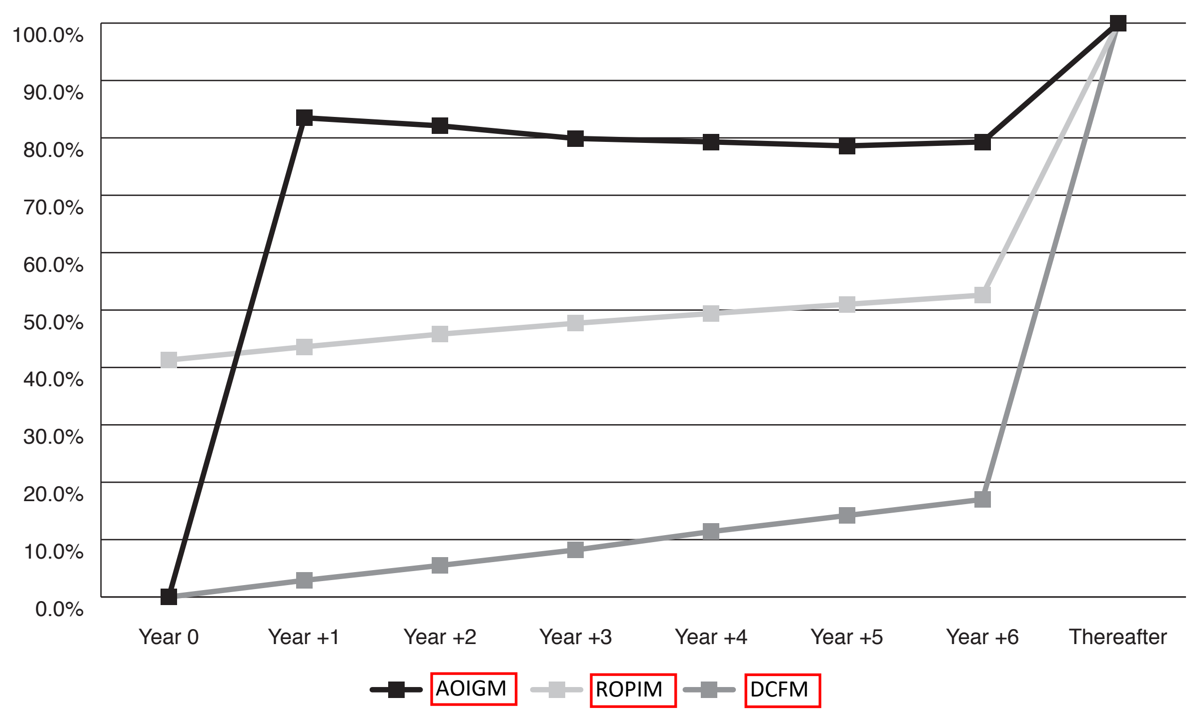
In this case, we see that, for PG, the free cash flow model has almost 83 percent of the value estimate coming from the periods beyond the forecast horizon; that is, 83 percent of the value estimate is the continuing value. Recall the discussion in Module 10 where we learned that the continuing value for the free cash flow model is the estimate of value at the horizon—we see that very little of our value estimate results from the explicit forecasts within the horizon but instead almost all of the estimate results from our forecast of the operating value at the horizon. If we don't know the value of the operations today, we are likely to have very little confidence in our forecasted value at the horizon. Of course!

For the residual operating income valuation model, anchoring on current *NOA*, which is known because it is available from PG's balance sheet, provides over 40 percent of our value estimate before we begin to use our forecasts. As a result, we do not rely as much on the forecast of the continuing value since it provides just under half of our value estimate (still a large number, but better).

When using the abnormal operating income growth model, we see that the portion of the estimated value coming from the periods beyond the explicitly forecasted horizon is only 20.1 percent. This is, by far, the lowest reliance on the continuing value. The degree to which the capitalized earnings (an element based on forecasts within the horizon) captures value in the abnormal operating income growth model will vary depending on the extent to which the forecasted earnings will be recurring rather than transitory.

To demonstrate the reliance on our forecasts, **Figure 13.1** shows how much of the total value estimate of $219,559 for PG's operations is captured as we add each year of the forecast period. This chart depicts the portion of value captured during the earlier years when our confidence in the forecasts is high and the later years when our confidence is low.

FIGURE 13.1 Percent of Value Captured as We Add Forecasted Years for PG



For PG, as long as we are willing to make a forecast of one-year ahead earnings, we have already captured over 80 percent of value. Thus, if we only are willing to forecast one year ahead, it would appear that the AOIGM is the model we would choose. Using one year of forecasts only provides less than 3 percent of the value for the DCF model! Note that the ROPIM, by anchoring on NOA, provides a considerable portion of value (for PG over 40 percent) before we even begin to forecast.

The take-away is that, if we are unsure of our short-term earnings estimates, have concerns about the weighted average cost of capital for operations used, or are working on a company for which it will take a long time to reach steady state, the accounting models clearly outperform the DCF model. From our examination of the various models in Modules 10, 11, and 12, we saw the following related to the anchors for valuation:

1. DCF does not have an anchor,
2. ROPIM anchors on book value (NOA_0) and often the subsequent forecasts within horizon earnings capture a lot of value, and
3. AOIGM anchors on capitalized earnings ($NOPAT_1/r_{WACC}$), which generally captures a lot of value.

These insights stem from the fact that *NOA* is the accountants' measure of value and *NOPAT* is the accountants' measure of value earned in the period. The practical choice of a valuation model requires one to question the confidence in the estimate obtained. In most cases, the answer to this question remains that we will be more confident within the horizon (i.e., before steady state is reached) than beyond (i.e., after steady state is reached).

When discussing models, one must also consider sources of information. Generally firms, analysts, and investors first consider the earnings of a company and from these they obtain forecasts of *FCF*. It is unclear when/if someone would directly forecast *FCF*s and then use them to determine implied earnings. Additionally, when users are intent on using the DCF valuation model, where *FCF* forecasts may, indeed, all be negative during the horizon (as in a growing firm, for example), they, essentially, will have to know the value at the horizon multiple years in the future to value the stock today!

CHOICE OF STEADY STATE GROWTH

The steady state growth rate is one of the most important choices in forecasting and implementing a value calculation. To make an intelligent choice, one must understand what it represents and its tie to the overall economy. Green, Hand, and Zhang [2016 *RAST*] found that 24 of 37 analyst reports, which explicitly stated their continuing (steady state) growth assumption, had a rate greater than 10 percent. As this rate is unlikely to exceed the long-term growth in the economy, these forecasted rates are unachievable; yielding a value estimate that will be overly optimistic!

In Module 10, we first introduced an assumption related to the continuing value by stating "The assumption that the rate of growth in free cash flows and the rate of growth in sales beyond the horizon are equal reflects that over time a firm's growth is driven by its ability to grow sales." The same point could be made with respect to growth in residual operating income and growth in abnormal operating income growth.

In this section, we underscore the point that, if implementation of the models is not done correctly and if steady state is not reached, the estimate of operating value will differ.[1] The common instructions to take the last explicit forecast, grow it, and then capitalize it are only appropriate if steady state has been reached. Essentially, when forecasting, we must forecast to a horizon, which is sufficiently far in the future to ensure that we are comfortable with the assumption that the forecasted growth rate is sustainable in the very long run. A frequently made assumption is that, eventually, the steady state growth rate will be equal to an assumed long-term economic growth rate.

During the beginning of this century, **Starbucks** (SBUX) was growing revenues at a very high rate. From the time that Starbucks went public in 1992, the firm's sales growth was greater than 20 percent until 2008! However, if you were looking at the company in the late 1990s or early 2000s, the assumption that the short-term growth in revenues (often exceeding 30 percent per year) would continue indefinitely would have been inappropriate. To value SBUX, like any firm, requires a sufficiently long forecast horizon to arrive at steady state. That is, forecasts for SBUX would need to extend far enough into the future that things have leveled off and the company is growing in line with economic growth.

In our forecasting for PG outlined in Module 7, arriving at a forecast mirroring economic growth for PG after five years did not appear to be too strong of an assumption, but to do so for a growing company like SBUX it would be inappropriate.

Consider SBUX during its period of growth prior to the 2008 economic downturn. Assume we are sitting in 2002 and making explicit full-information forecasts for 2003 to 2007. By looking at its actual results for the years ending in September 2003 to 2007, we can see that perfect-foresight forecasts would be the following estimates of sales growth, *PM*, and *ATO* shown in **Exhibit 13.4**:

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[1] This reminds us of the reason why additional information was included in Module 4 and adjustments were made in Module 5. We wanted to ensure that we did not mistakenly think we had reached steady state when we had not and we wanted to ensure that the forecasted values going forward were not incorrect due to misunderstanding current information.

EXHIBIT 14.2 Updating PG's *NFL* to December 5, 2015

Amounts in millions	June 30, 2015	Unchanged	Sept. 30, 2015 Form 10-Q Balance	Sept. 30, 2015 Form 10-Q Fair Value
Financial Assets (*FA*)				
Cash and cash equivalents	$ 5,329	$ 5,329		
Available-for-sale investment securities	4,767		$ 4,901	
Deferred Tax Assets:				
Pension and postretirement benefits	1,839	1,839		
Stock-based compensation	2,398	2,398		
Accrued interest and taxes	281	281		
Noncurrent pension assets	10,605	10,605		
Noncurrent other postretirement assets	3,470	3,470		
	28,689			
Financial Liabilities (*FL*)				
Accrued liabilities:				
Pension benefits	(39)	(39)		
Other postretirement benefits	(20)	(20)		
Liabilities held for sale	(1,187)		(2,222)	
Current portion of long-term debt	(2,772)			
Commercial paper	(8,807)		(13,093)	
Other debt due within one year	(442)			
Long-term debt	(18,329)			$(19,384)
Long-term capitalized lease liability	(1,438)	(1,438)		
Stock based compensation liability	(3,956)	(3,956)		
Other noncurrent liabilities:				
Pension benefits	(15,912)	(15,912)		
Other postretirement benefits	(4,884)	(4,884)		
Uncertain tax positions	(1,016)	(1,016)		
Convertible class A preferred stock	(1,077)		(1,067)	
Non-controlling interest	(631)		(667)	
	(60,510)			
		(3,343)	(12,148)	(19,384)
Net Financial Liabilities (*NFL*)	$(31,821)		$(34,875)	

The difference between operating value and equity value is the value of the firm's financing activities. After reformulating the financial statements, the calculation of equity value is quite straightforward; we subtract the estimate of the fair value of *NFL* of $27,647 from our operating value of $219,559 to arrive at an estimated equity value of $191,912. Further adjustments to this estimate are necessary due to issues related to the dates of the valuation, but before examining these adjustments later in this module, let us consider the direct valuation of equity based on the valuation models shown in Modules 10, 11, and 12.

VALUATION FORMULAS FOR EQUITY

Beginning in Module 10, we focused on the cash flows to the debt and equity holders. Some refer to these as "free cash flows of the firm" to distinguish them from the "free cash flows to equity," which is the portion of free cash flows that will go to the equity holders. The free cash flows to equity are the payoffs that the equity holders receive. More commonly, we refer to the payoffs to equity holders as dividends.

In Module 10, we valued the operations based on the payoffs expected to be made to the debt and equity holders. We now show the formula that may be used to value the firm based on expected payoffs to be made to the equity holders. That is, we show the formula based on dividends to equity holders: